Fractions: Developing Effective Instruction
Recommendations from a What Works Clearinghouse Practice Guide

2011 NCSM Summer Leadership Academy

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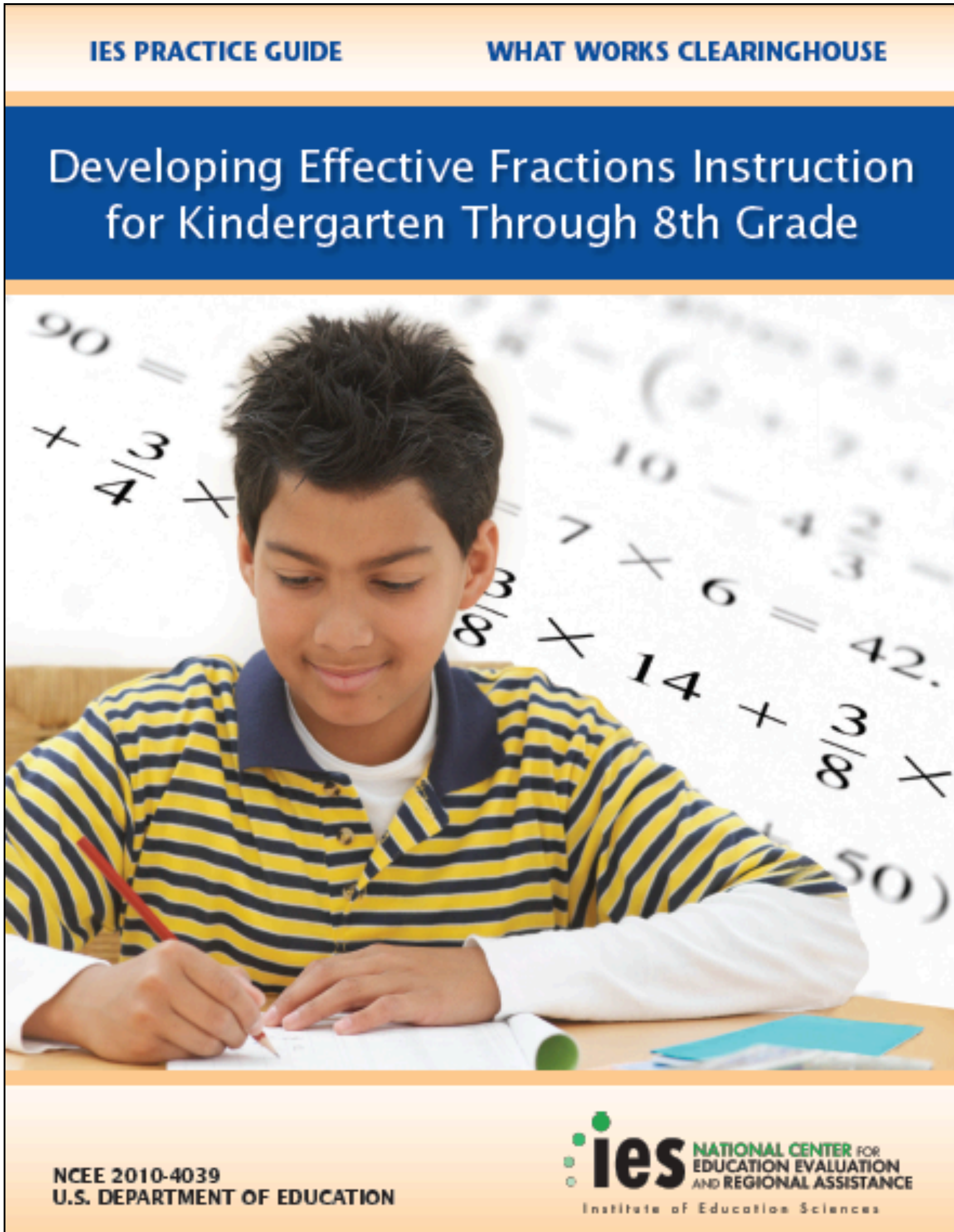
HAS THE SCHOOL BUS COME YET?
OBVIOUSLY NOT..WE'RE STILL HERE, AREN'T WE?
I WAS AFRAID I MIGHT HAVE ALREADY GOTTEN ON IT, RODE TO SCHOOL, STAYED THERE ALL DAY, RODE BACK, GOT OFF AND WAS STANDING HERE FORGETTING TO GO HOME..
YESTERDAY WE STARTED FRACTIONS..
THAT'LL DO IT..



Panel

Robert Siegler (Chair)
Carnegie Mellon University

Thomas Carpenter
University of Wisconsin-Madison

Francis (Skip) Fennell
McDaniel College

David Geary
University of Missouri at Columbia

James Lewis
University of Nebraska-Lincoln

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Staff

Jeffrey Max
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Mathematical Policy Research

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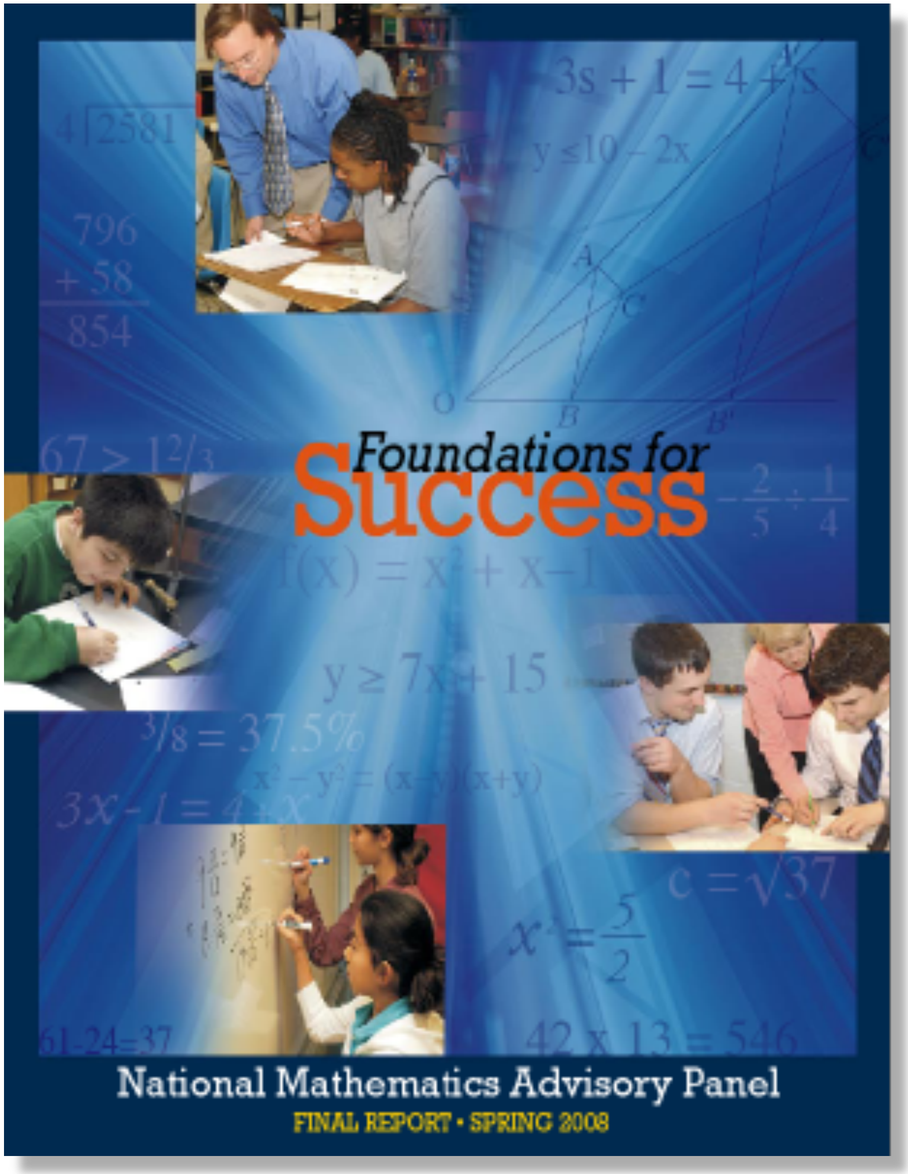
Institution of Education Sciences (IES)

How did this get started…?

Fraction issues…

- Conceptual Knowledge and Skills
- Learning Processes
- Assessment
- Survey of Algebra Teachers



How did this get started…? NMAP - Student Preparation

The first question concerned the adequacy of student preparation coming into the Algebra I classes. The topics that were rated as especially problematic were:

- Rational numbers;
- Solving word problems, and;
- Basic study skills.

Final Report on the National Survey of Algebra Teachers for the National Math Panel, NORC, September, 2007

Making Sense of Numbers…

1. Ability to compose and decompose numbers…
2. Ability to recognize the relative magnitude of numbers – including comparing and ordering.
3. Ability to deal with the absolute magnitude of numbers – realizing, for instance there are far fewer than 500 people in this session!
4. Ability to use benchmarks.
5. Ability to link numeration, operation, and relation symbols in meaningful ways.
6. Understanding the effects of operations on numbers.
7. The ability to perform mental computation through invented strategies that take advantages of numerical and operational properties.
8. Being able to use numbers flexibly to estimate numerical answers to computations, and to recognize when an estimate is appropriate.
9. A disposition towards making sense of numbers.

"It is possible to have good number sense for whole numbers, but not for fractions…"

Sowder, J. and Schappelle, Eds., 1989

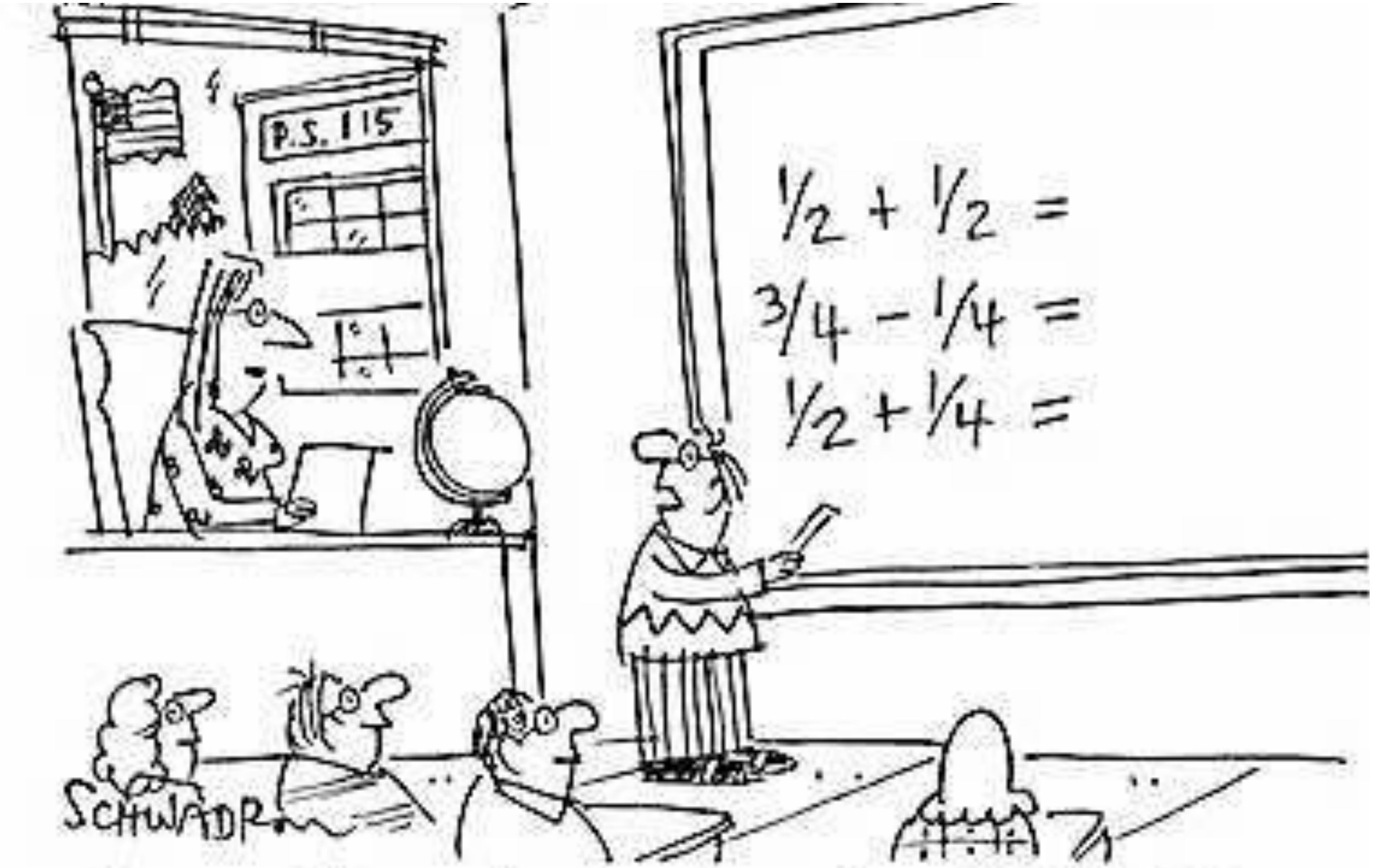
Why are fractions so difficult?

American students' weak understanding of fractions

- 2004 NAEP - 50% of 8th-graders could not order three fractions from least to greatest (NCTM, 2007)
- 2004 NAEP, Fewer than 30% of 17-year-olds correctly translated 0.029 as 29/1000 (Kloosterman, 2010)
- One-on-one controlled experiment tests - when asked which of two decimals, 0.274 and 0.83 is greater, most 5th- and 6th-graders choose 0.274 (Rittle-Johnson, Siegler, and Alibali, 2001)
- Knowledge of fractions differs even more between students in the U.S. and students in East Asia than does knowledge of whole numbers (Mullis, et al., 1997)

“[**Proficient** fourth graders] should have a <u>conceptual understanding</u> of fractions and decimals…”

(NCES, 2009, p. 18).



"MY DAD SAYS FRACTIONS ARE OBSOLETE SINCE THE STOCK MARKET MOVED TO DECIMALS."

Stock Exchanges Trade Fractions for Decimals

Stock Exchanges Swap
16ths for Decimals

By Roland Jones
Aug. 23

abc NEWS

Post a Comment

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The French have done it for hundreds of years and the British embraced it decades ago. But when it comes to quoting stock prices, the U.S. markets have avoided trading shares in decimals for more than 200 years.

Instead, stock prices, like weights and measures, haven't changed since the early 1800s when stockbrokers in New York and Philadelphia happily traded securities in fractions, like eighths and sixteenths of dollars.

That's about to change. On Monday, 13 stocks on the New York Stock Exchange and their associated options will start trading in dollars and cents.

ABC News, 8/23/2000 http://abcnews.go.com/Business/story?id=89457&page=1

Facets of the lack of student conceptual understanding… just a few

- Not viewing fractions as numbers at all, but rather as meaningless symbols that need to be manipulated in arbitrary ways to produce answers that satisfy a teacher
- Focusing on numerators and denominators as separate numbers rather than thinking of the fraction as a single number.
- Confusing properties of fractions with those of whole numbers

IES – Practice Guide - Fractions, 2010 (pp. 6-7)

Fractions Practice Guide authors concluded:

"A high percentage of U.S. students lack conceptual understanding of fractions, even after studying fractions for several years; this, in turn, limits students' ability to solve problems with fractions and to learn and apply computational procedures involving fractions."

IES – Practice Guide - Fractions, 2010 (pp. 6-7)

Research – Another Look

- Whole Number Concepts and Operations
  - Citations: 334
- Rational Numbers and Proportional Reasoning
  - Citations: 140
- In the 2000's: only 9 citations;
    - 109 in Whole Number Concepts and Operations
    - 1/12th

NCTM, 2007; 2nd Handbook – Research on Mathematics Teaching and Learning

- “The number of references in this chapter predating 1992 is far greater than the number appearing since the last handbook.”

- “This crisis…stems from:
  - Teachers are not prepared to teach content other than part-whole fractions;
  - Long-term commitment is needed because rational number topics are learned over many years.
  - The nonlinear development of the content does not mesh well with scope and sequence currently prescribing mathematics instruction in schools; and
  - In comparison to a domain such as early addition and subtraction, little research progress is evident.”

Lamon, 2007 in 2nd Handbook Research on Mathematics Teaching & Learning

Think about…
Curriculum, Assessments,
Research…

IES* Practice Guide

- The research base for the guide was identified through a comprehensive search for studies over the past 20 years that evaluated teaching and learning about fractions.
- The process yielded more than 3,000 citations. Of these, 132 met the What Works Clearinghouse criteria for review (4%), and 33 met the causal validity standards of the WWC (1+%).

*Institute of Education Sciences

Levels of Evidence

- **Strong** - positive findings are demonstrated in multiple well-designed, well-executed studies, leaving little or no double that the positive effects are caused by the recommended practice
- **Moderate** - well-designed studies show positive impacts, but some questions remain about whether the findings can be generalized or whether the studies definitively show that the practice is effective.
- **Minimal** - data may; suggest a relationship between the recommended practice and positive outcomes, but research has not demonstrated that the practice is the cause of positive outcomes.

IES – Practice Guide - Fractions, 2010 (See Table 1 on pp. 4-5)

What’s in a “Practice Guide?” (On your flash drive!)

- Review of recommendations (p. 1)
- Levels of evidence (pp. 3-5)
- Introduction (p. 6)
- Audience and grade level (p. 7)
- Recommendations, “Roadblocks” (e.g., p. 18) and Suggestions (pp. 12-46)
  – Example: **Roadblock 1.3** (p. 18) - *When creating equal shares, students do not distinguish between the number of things shared and the quantity shared (confusing equal numbers of shares with equal amounts of shared).* **Suggested Approach**…
- Glossary (p. 47)
- Appendices, endnotes, index, references, etc.(pp. 49-84)

Recommendations

1. Build on students' **informal understanding of sharing and proportionality** to develop initial fraction concepts. **(Minimal)**
2. Help students recognize that **fractions are numbers and that they expand the number system beyond whole numbers**. Use **number lines as a central representational tool** in teaching this and other fraction concepts from the early grades onward. **(Moderate)**
3. Help students **understand why procedures for computations** with fractions **makes sense**. **(Moderate)**
4. Develop students' **conceptual understanding** of strategies for solving **ratio, rate, and proportion problems before exposing them to cross-multiplication** as a procedure to use to solve such problems. **(Minimal)**
5. Professional development programs should place a high priority on improving teachers' **understanding of fractions** and **how to teach them**. **(Minimal)**

IES – Practice Guide - Fractions, 2010

Recommendation 1

Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.

– Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole objects.

– Extend equal-sharing activities to develop students' understanding of ordering and equivalence of fractions.

– Build on students' informal understanding to develop more advanced understanding of proportional reasoning concepts. Begin with activities that involve similar proportions, and progress to activities that involve ordering different proportions.

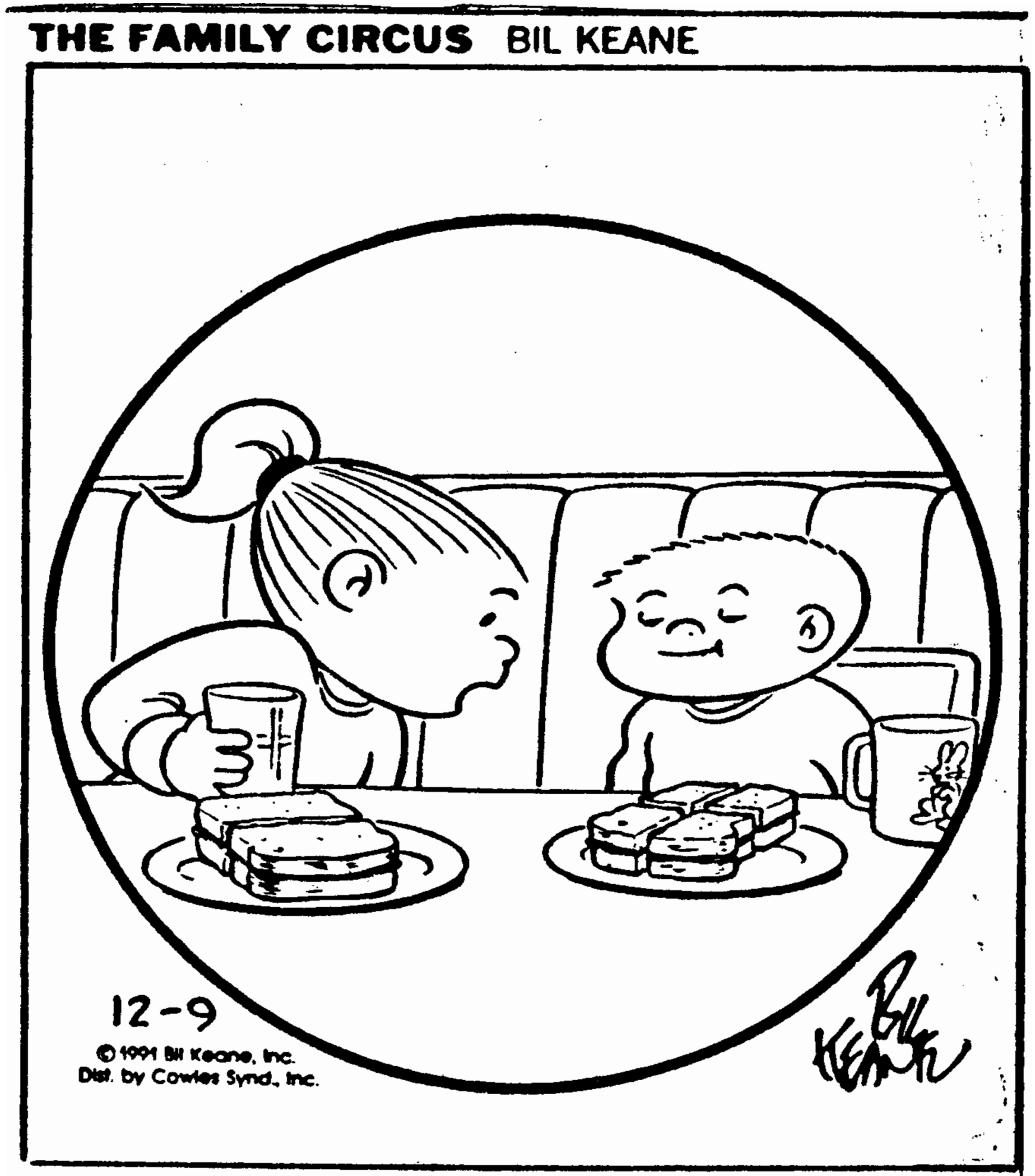
livescribe

Elena and her 3 friends ate 9 cookies.
How many cookies did each friend eat?

Play/Pause

0:00 / 2:09

Fair share: Another Interpretation



"How come PJ got four sandwiches and I only got two?"

- How can we share **eleven** hoagies (aka subs) among four people?

- How can we share **eleven** hoagies (aka subs) among five people?

Adapted from Fosnot and Dolk

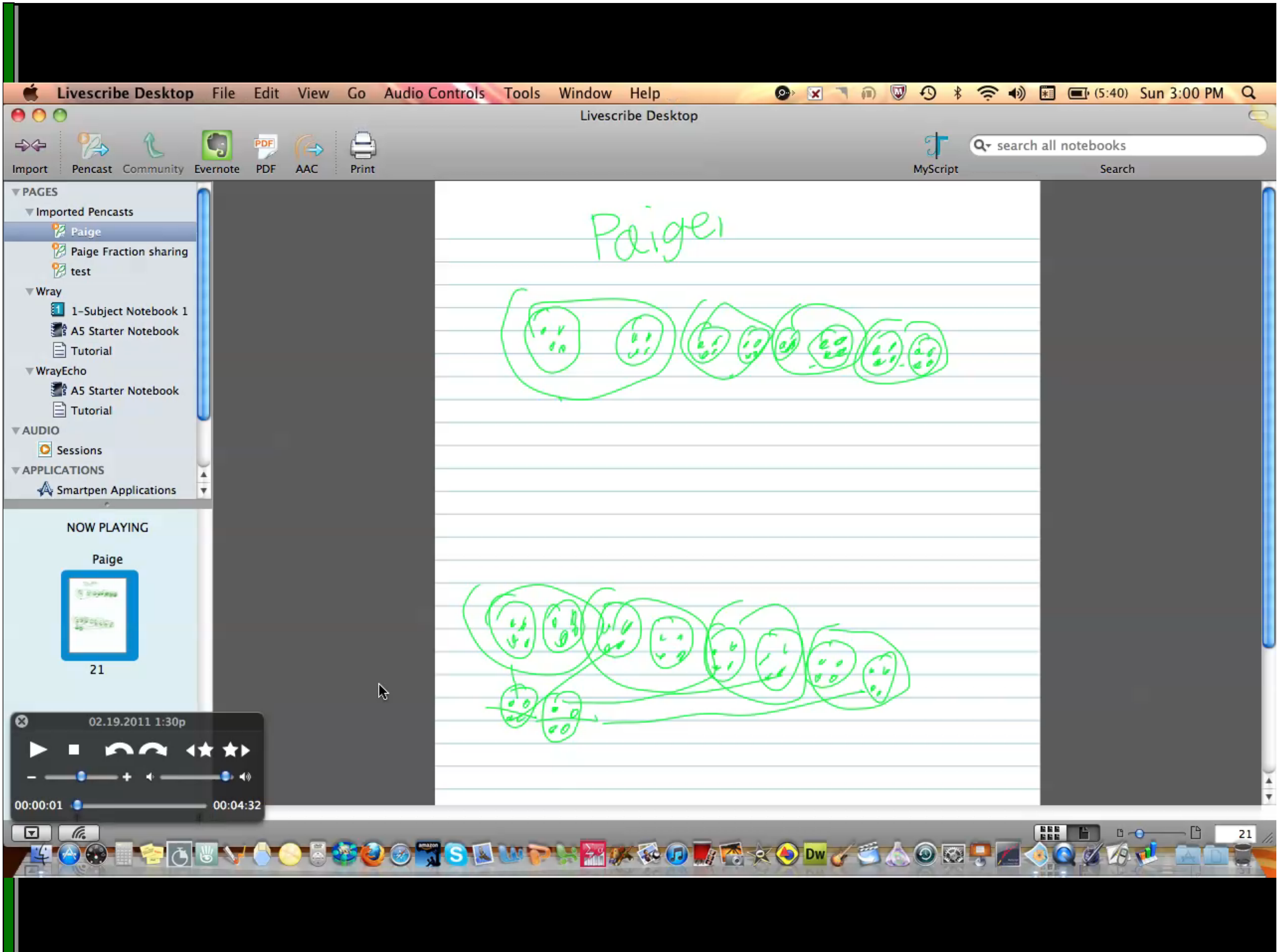
How about if we have six people and we need to share 5 cookies?*

Division involving equal shares is a process that many understand intuitively.

*food seems to work – a lot!

Recommendation 1

1. <u>An early example</u>: If we cut this cake so that you and two friends could share it, what would the slices look like? How can we talk about and write how much of the cake you will each get? (Circular or rectangular regions)
2. Would you rather share your favorite pizza with 3 other people or with 7 other people? (McNamara and Shaughnessy, ***Beyond Pizzas and Pies***, 2010)
3. How can you share 8 cookies with the four children in your family? Can you make a drawing to show me how you would do this? (Tiles or counters)
   – **Extending** - What if you had 10 cookies; now how would you share them with the 4 children in your family?
4. If you had 13 cookies to share among 4 friends, how many cookies would each person get? Would this be *more* or *less* than if you shared 12 cookies? (Tiles or counters)



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AUDIO
Sessions
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Smartpen Applications
NOW PLAYING
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Recommendation 1

Think About

5. What about 6 people sharing 5 cookies? How much for each person?
6. Sharing 12 cards with 5 people; Sharing 5 cards with 12 people – how are these approached differently?
7. Each delivery is 3 miles. There were 7 deliveries. How many miles were traveled? *How would you represent this?*
8. Connections to CCSS (Grades 1, 2, 6, and 7)

Survey

- Brought your cell phone? *Take it out!*

Recommendation 2

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

- Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share.
- Provide opportunities for students to locate and compare fractions on number lines.
- Use number lines to improve students' understanding of fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions.
- Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students' ability to translate among these forms.

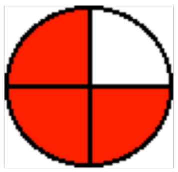
14. Name a fraction between:

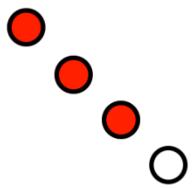
0 and 1 $\frac{1}{2}$

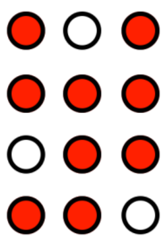
$\frac{1}{4}$ and $\frac{1}{2}$ $\frac{1}{3}$

$\frac{5}{6}$ and 1 I dont know

Thinking about ¾…

a) 

b) 

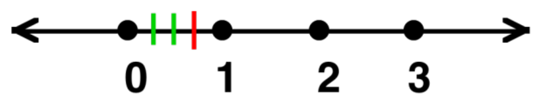
c) 

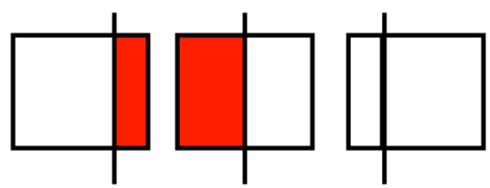
d) How many 4's are there in 3?

e) 18 crayons out of a box of 24

f) .75

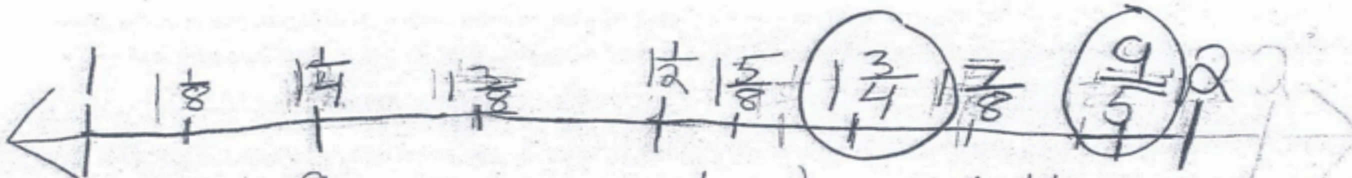
g) I want to share 3 bottles of soda equally among 4 people. How much will each person get?

h) 

i) 

1) Draw a number line and show where to place the fraction 9/5.
Explain your thinking.

$1\frac{75}{100}$ $1\frac{80}{100}$



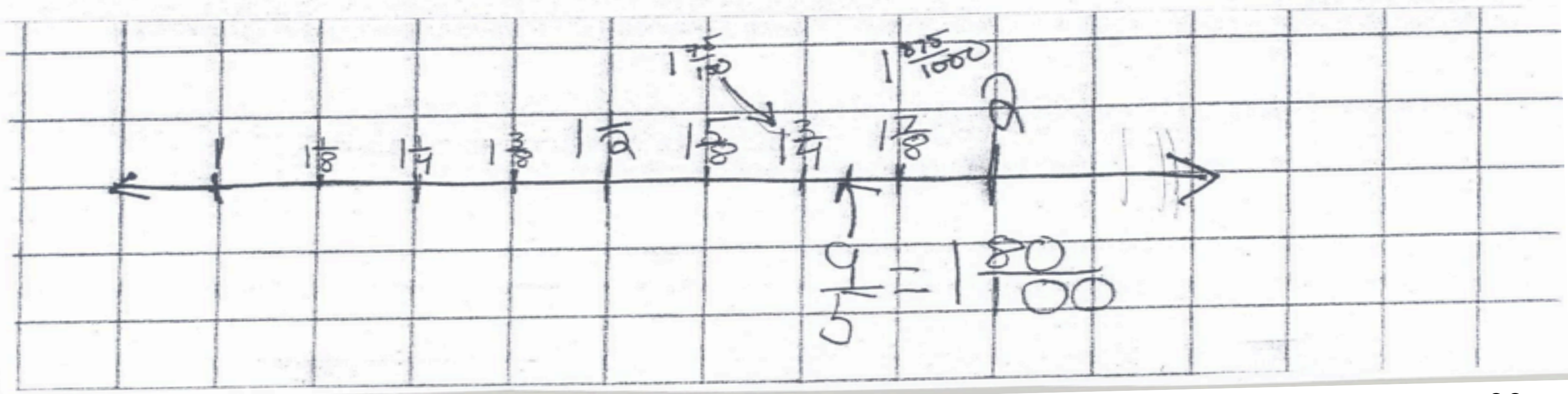
$\frac{9}{5}$ is equivlant to $1\frac{4}{5}$ and is almost 2 so it has to go there.

$\frac{7}{8} = \frac{87.5}{100}$

2) Order from smallest to greatest: 7/8, 3/8, 5/8, and 9/8.

$\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$ $\frac{9}{8}$

$\frac{1}{8} = 12.5$



1) Draw a number line and show where to place the fraction 9/5. Explain your thinking.

Because 9/5 is a topheavy fraction I said it was = to 1 4/5 ($\frac{9}{5}$, 9-5 = $\frac{4}{5}$ 1 4/5). 1 4/5 is right behind 2 on the # line.

2) Order from smallest to greatest: 7/8, 3/8, 5/8, and 9/8.

$\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{9}{8}$

3) Order from smallest to greatest: 3/5, 3/7, 3/4, and 3/8.

$\frac{3}{8}$, $\frac{3}{7}$ $\frac{3}{5}$ $\frac{3}{4}$

$\frac{3}{4}$

- What happens to the value of the fraction if the numerator is increased by 1?
- What happens to the value of the fraction if the denominator is decreased by 1?
- What happens to the value of the fraction if the denominator is increased?

Ordering Fractions

Write these fractions in order from least to greatest. Tell how you decided.

• 5/3	5/6	5/5	5/4	5/8
• 7/8	2/8	10/8	3/8	1/8

You can’t make this stuff up!

- The weather reporter on WCRB (a Boston radio station) said there was a 30% chance of rain. The host of the show asked what that meant.
- The weather reporter said, ``It will rain on 30% of the state.''
- ``What are the chances of getting wet if you are in that 30% of the state?''
- ``100%.''

Recommendation 2

1. Using bar diagrams can you show 3/4 and 7/8? Which is greater? How do you know? (It’s okay to use circular or rectangular regions or the number line or double number lines.)
2. Use any of the materials (perhaps all of them!) to represent the following equivalent fractions:
   - 1/2 = ?/6
   - 2/3 = ?/6
   - One additional equivalent pair
3. Place 1/8, 5/8, 7/8, and 9/8 on your number line. Which is the smallest fraction? The largest? How do you know?
   - **Extend**: Now place 2/3, 2/8, 2/2, 2/10, and 2/1 on your number line. Which fraction is smallest? Largest? How do you know?
4. If you could have any of the following amounts of your favorite food at a party, which would you rather have and why? Use drawings, bar diagrams, or other tools to help you in your thinking.
   0.5, 60%, 7/8, 1/2, 0.78 or 0.01

Recommendation 2

Think About

4. How can you represent 5 ⅔? Use any representation you like. How about 17/4? Use any representation you like.
   - **Extend**: Is one representation model easier to “see” than others?
5. If you wanted to establish that 0.9 > 7/8, how would you do that?
6. How could you establish that 12/4 = 24/8 = 6/2?
7. Using your number line, establish 0 and 1 and then 1/2 and continue to halve fractions until you run out of space. What’s the point of all of this?
8. When do the materials show their limitations with equivalence?
9. Why is the number line described as “under-utilized” as a representation tool?
10. Connections to CCSS (Grades 3, 4, 5)

Survey

- Cell phone - *take it out!*

Recommendation 3

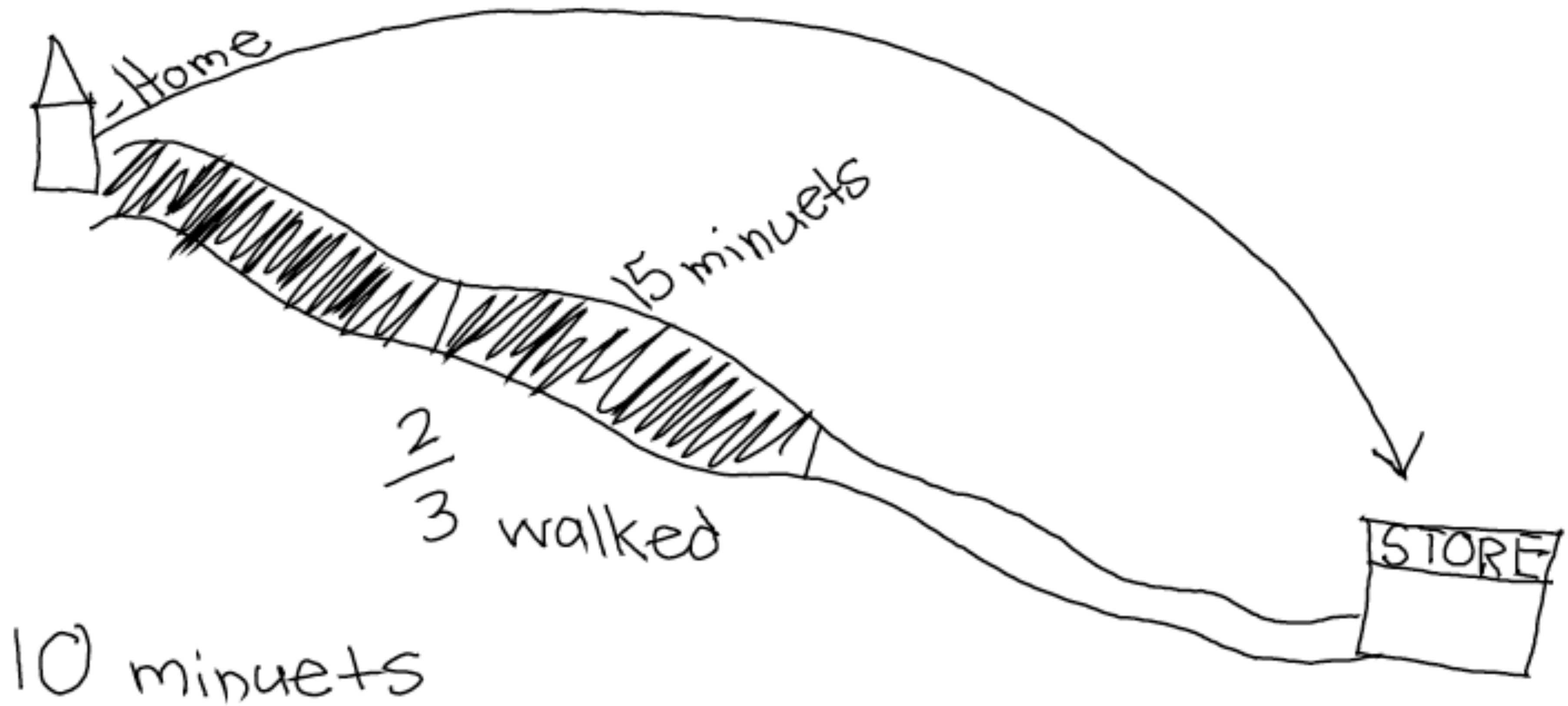
Help students understand why procedures for computations with fractions makes sense.

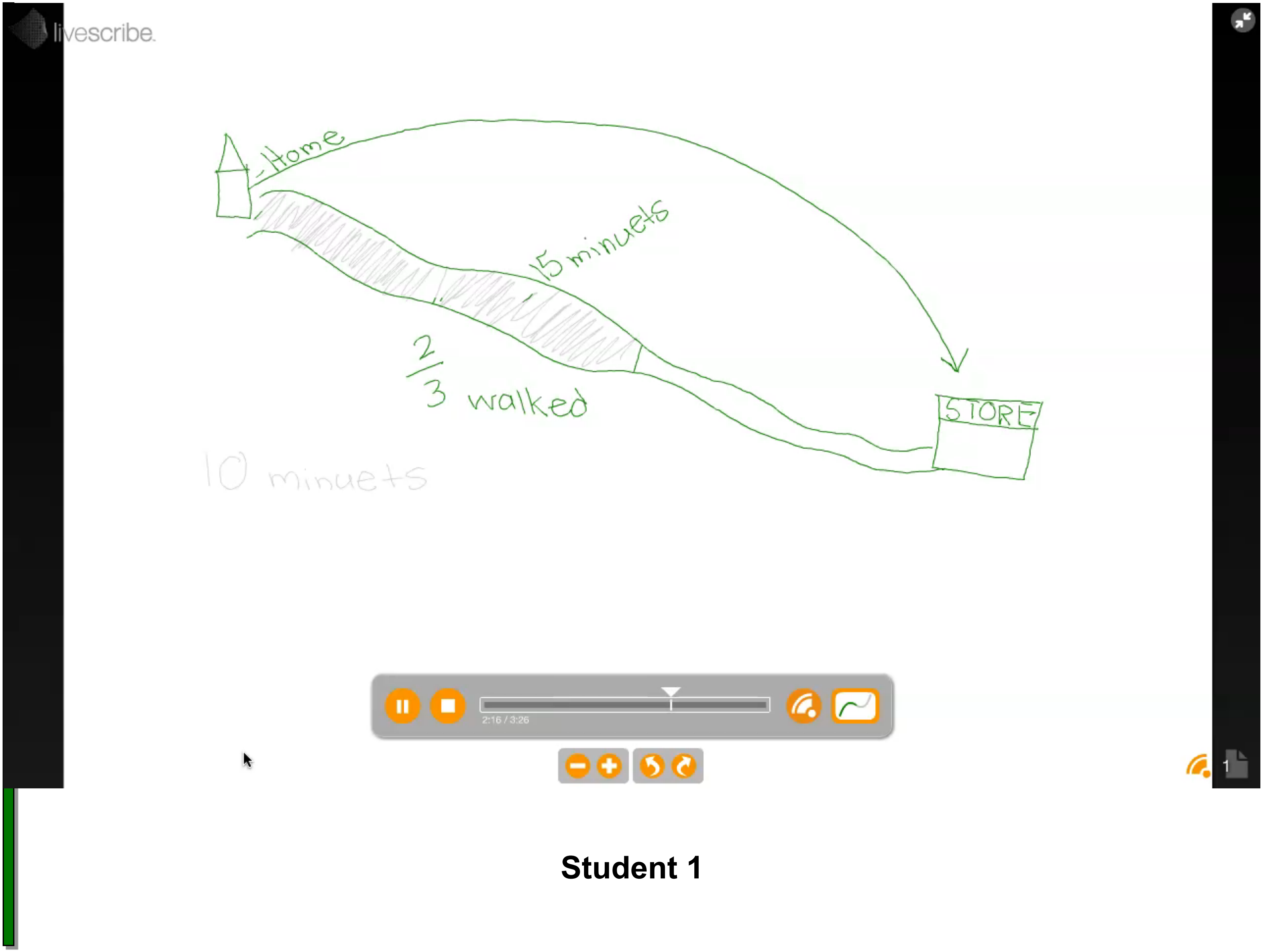
- Use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures.
- *Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions.*
- *Address common misconceptions regarding computational procedures with fractions.*
- Present real-world contexts with plausible numbers for problems that involve computing with fractions.

The walk from home to the store takes 15 minutes. When Bill asked Ming how far they had gone, Ming said that they had gone 2/3 of the way.

How many minutes had Bill and Ming walked? (Assume constant walking rate.)

Student 1



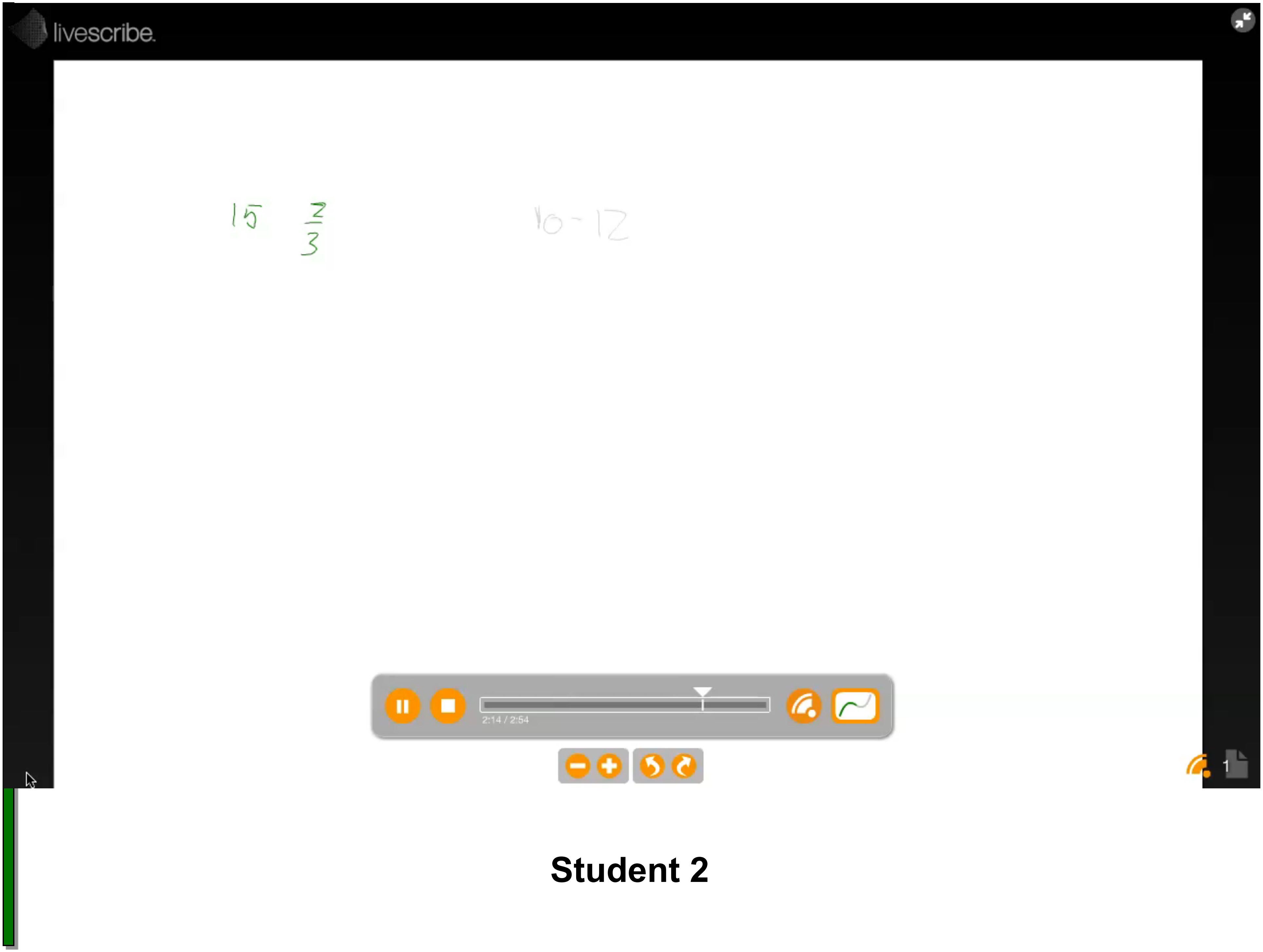

livescribe.
Home
15 minuets
2/3 walked
STORE
10 minuets
2:16 / 3:26
1

Student 1

Student 2

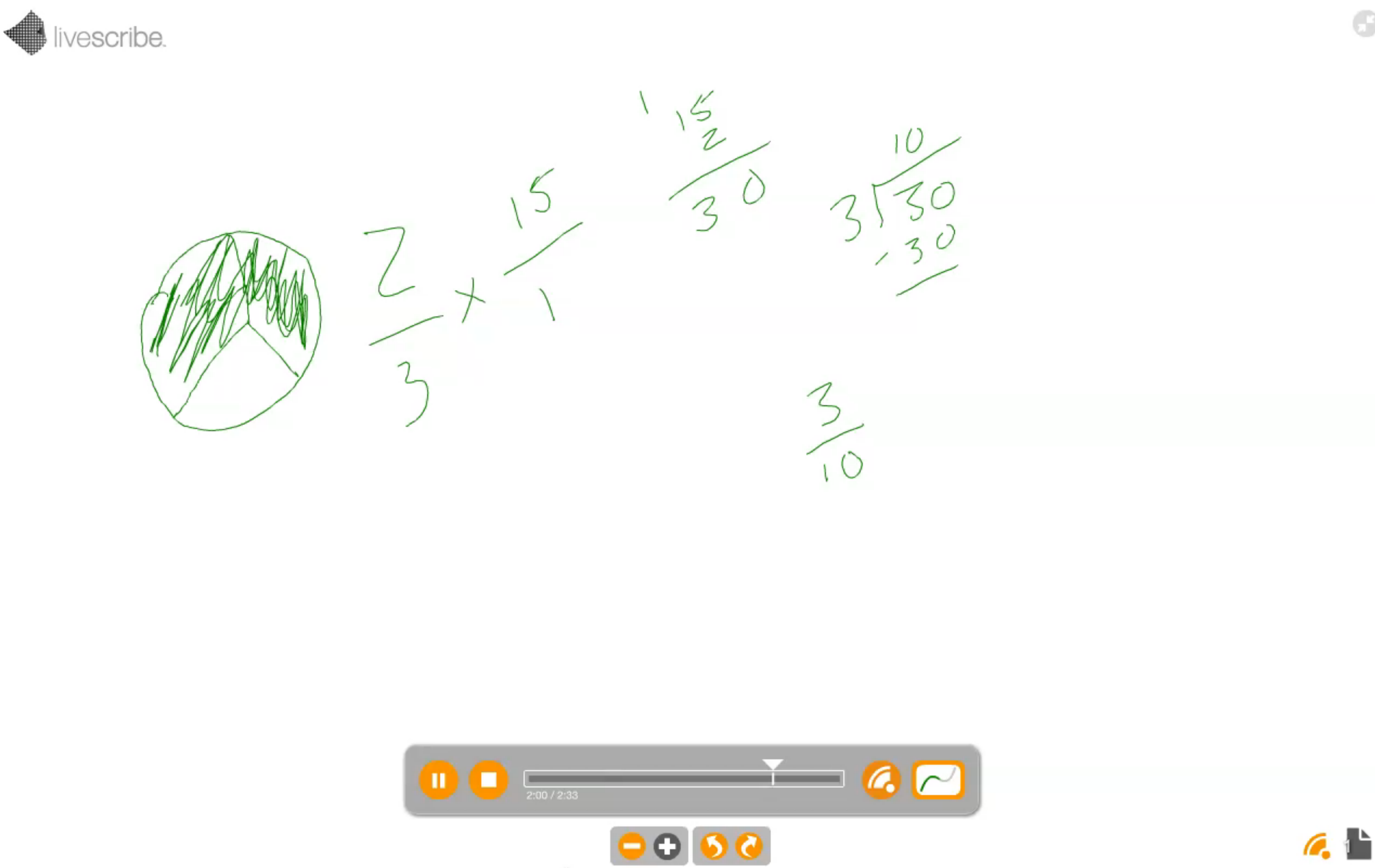
15 $\frac{2}{3}$

10 - 12

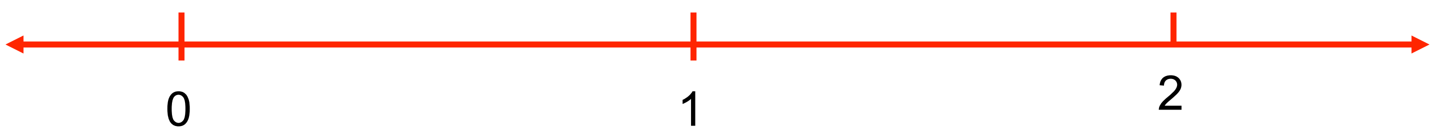


Student 2

The walk from home to the store takes 15 minutes. When Bill asked Ming how far they had gone, Ming said that they had gone 2/3 of the way. How many minutes had Bill and Ming walked? (Assume constant walking rate.) (**Grade 8 Student**)



- Tell me about where 2/3 + 1/6 would be on this number line (Cramer, Henry, 2002).



Sense Making:

“2/3 is almost 1, 1/6 is a bit more, but the sum is < 1”

7/8 – 1/8 = ?

- Interviewer: Melanie these two circles represent pies that were each cut into eight pieces for a party. This pie on the left had seven pieces eaten from it. How much pie is left there?
- **Melanie:** ***One-eighth, writes 1/8*.**
- Interviewer: The pie on the right had three pieces eaten from it. How much is left of that pie?
- **Melanie:** ***Five-eighths, writes 5/8.***
- Interviewer: If you put those two together, how much of a pie is left?
- **Melanie:** ***Six-eighths, writes 6/8.***
- Interviewer: Could you write a number sentence to show what you just did?
- **Melanie:** ***Writes 1/8 + 5/8 = 6/16.***
- Interviewer: That's not the same as you told me before. Is that OK?
- **Melanie:** ***Yes, this is the answer you get when you add fractions.***

What Happens Here?

- 1/2 x 3/4 < or > 3/4
- 3/4 x 1/2 < or > 1/2

- 1/2 ÷ 3/4 < or > 1/2
- 3/4 ÷ 1/2 < or > 3/4

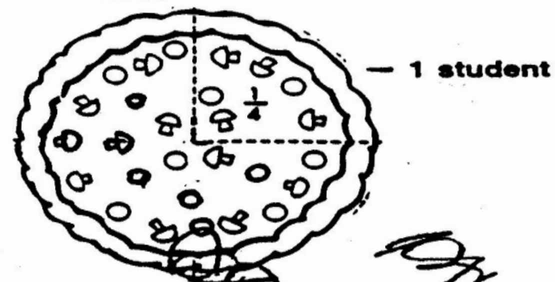
Now what?

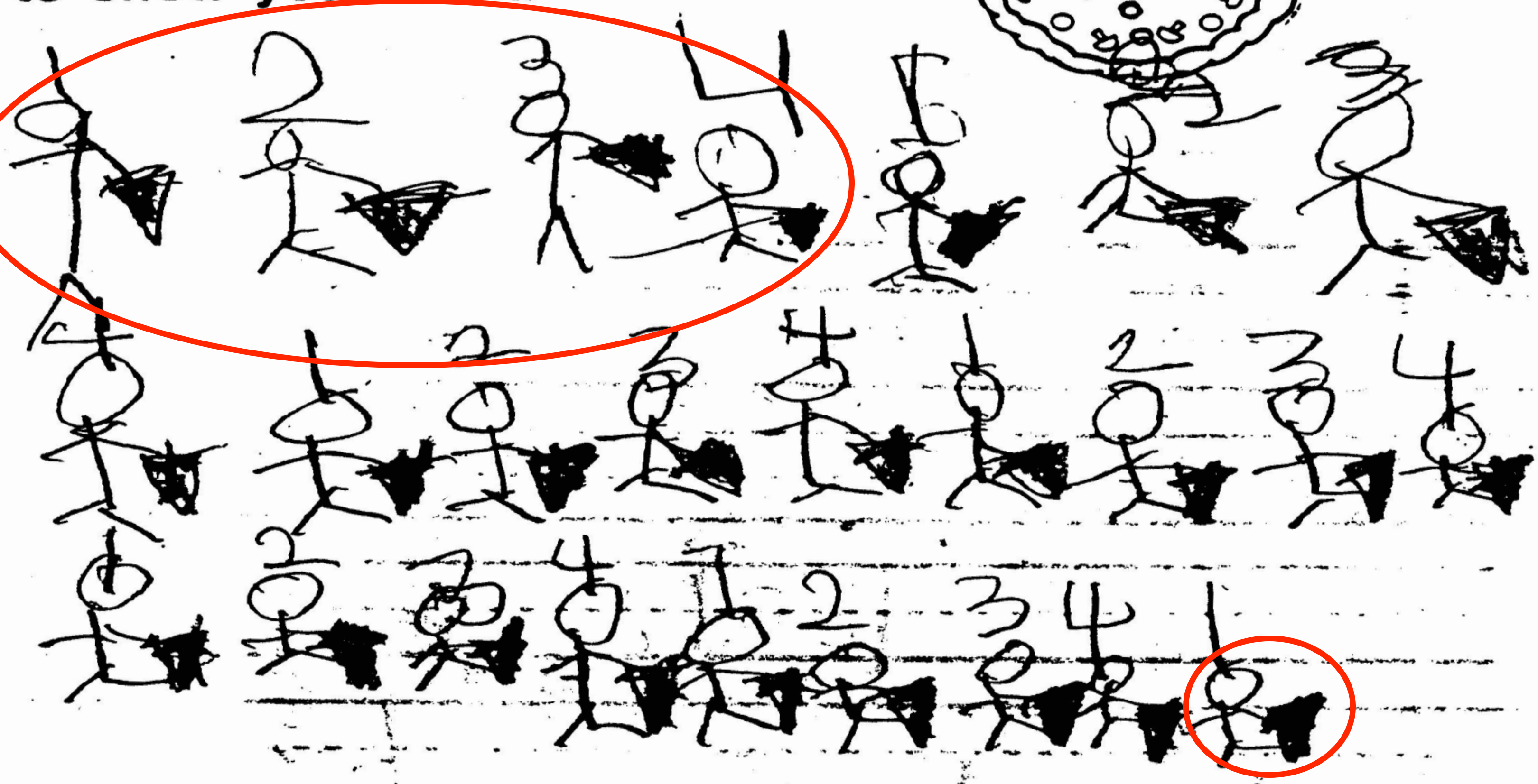
- There are 25 students in our class. Each student will get ¼ of a pizza. Your job is to find out how many pizzas we should order. Be sure to show your work.

- How many pizzas should we order?

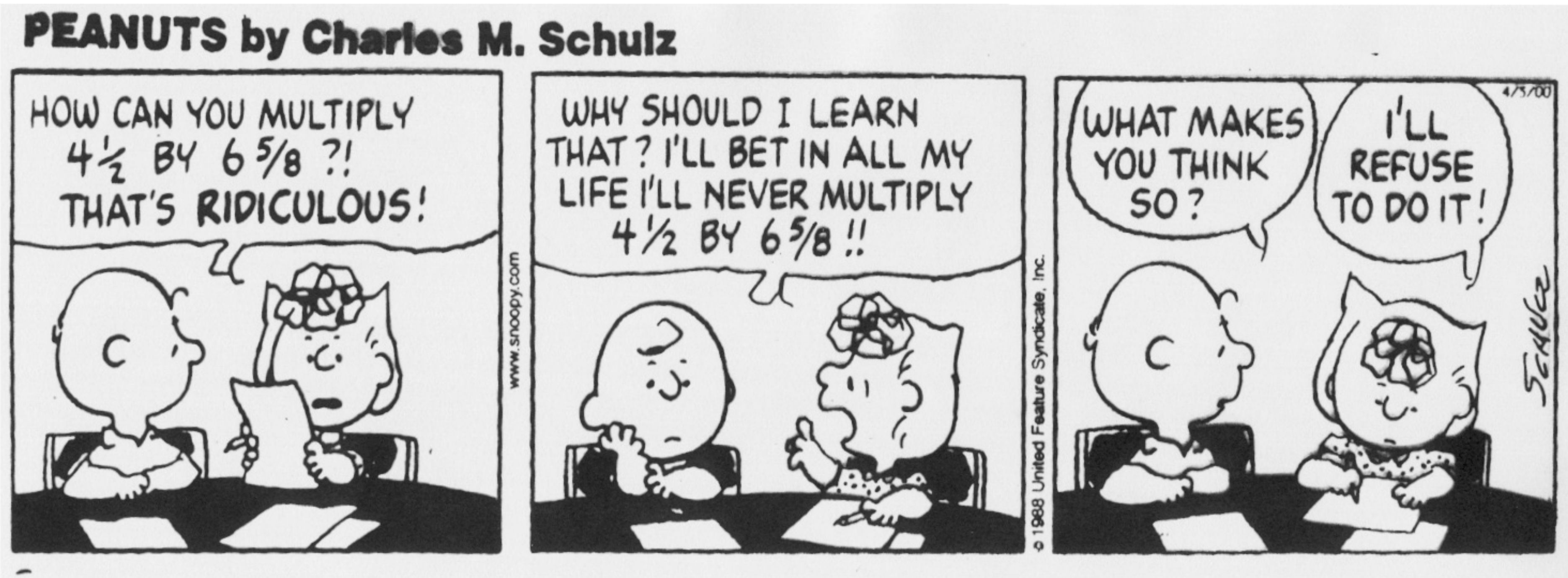
Task 1

There are 25 students in our class. Each student will get $\frac{1}{4}$ of a pizza. Your job is to decide how many pizzas we should order? Be sure to show your work.





How many pizzas should we order? 7


PEANUTS by Charles M. Schulz
HOW CAN YOU MULTIPLY 4 1/2 BY 6 5/8 ?! THAT'S RIDICULOUS!
www.snoopy.com
WHY SHOULD I LEARN THAT? I'LL BET IN ALL MY LIFE I'LL NEVER MULTIPLY 4 1/2 BY 6 5/8 !!
© 1988 United Feature Syndicate, Inc.
WHAT MAKES YOU THINK SO?
I'LL REFUSE TO DO IT!
SCHULZ

Recommendation 3

1. Use a rectangular area model to show 1/4 + 1/2 (use tiles or rectangular regions). Could you show this on a number line? What about 3/8 and ½?
2. Use a number line to show your solution to the following problem. Nita walked 1/2 mile on Monday, Wednesday, and Friday. On Saturday she walked 3/4 of a mile. What was her mileage for the 4 days?
3. Use a double number line or bar diagram to represent the solution to the following problem. Brett had 3 of 4 cards in one set and Stacey had 10 of 12 cards in another set of cards. Who had the greater proportion of their deck of cards? (Note: clock)
4. The class book of records claims that Cam eats 1/2 of an energy bar everyday. Did he eat more or less than 6 energy bars in a week? Make a drawing to support your answer.
5. Mia’s mother had 2/3 of Mia’s cake left. Mia decided to eat 1/2 of what was left. How much of the cake did she eat? Provide a drawing to show what Mia ate.
6. Cooper decided to walk 2 ½ miles on the trail. After each 1/2 mile he stopped for a sip of water. How many times did he stop? Show your solution using a number line.

Recommendation 3

Think About

7 How can you show that 1/2 + 1/4 is < 0.80?

8. If fractions and decimals are more carefully linked, the common misconception of 0.6 x 0.7 = 4.2 could be explained through representing the example as 6/10 x 7/10 = 42/100 or 0.42.

9. We would want students to mentally visualize solutions for:
   - 1/2 x 1/4
   - 1/2 ÷ 1/4
   - Can you visualize “mental number lines” and other representations for these examples?

10. Emphasis on estimation, and linking 3/4, 0.75, and 75%.

11. When do the materials show their limitations with operations?

12. Connections to CCSS (Grades 3, 4, 5, 6)

Recommendation 4

Develop students' understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.

– Develop students' understanding of proportional relations before teaching computational procedures that are conceptually difficult to understand (e.g., cross-multiplication). Build on students' developing strategies for solving ratio, rate, and proportion problems.

– Encourage students to use visual representations to solve ratio, rate, and proportion problems.

– Provide opportunities for students to use and discuss alternative strategies for solving ratio, rate, and proportion problems.

Lakers vs Nuggets

- Which player from the Lakers had the best shooting percentage?
- Which player from the Lakers had the worst shooting percentage?
- Same items for Nuggets
- Which players scored the most points, etc.?

You can't make this stuff up

- Gettysburg Outlets – July 3, 2009. 50% off sale on all purchases at the Izod store. Sign indicates 50% off the all-store sale.
  - Patron – "well that means it's free."
  - Clerk – "no sir, it's 50% off the 50% off sale."
  - Patron – "well, 50% + 50% is 100% so that means it should be free."
  - This went on for a while. AND, there was a sign indicating 70% off for some items, meaning 70% off the 50% off original sale, which our patron would interpret as the item being free and 20% in cash!

- On a scale 1” = 12 miles. If two places are 4” apart, how far are they away from each other in miles?

1”	12 miles
4”	


Get Your Pennies Worth
The Produce Place's
9¢ Sale
A Delightful Snack. 117 Size
Kiwi
Fruit
ea. .9
Great For Eating or Juice. 80 Size
Florida Valencia
Oranges
ea. .9
Cut Up in A Salad
Kirby
Pickles
ea. .9
Add Flavor to Your Favorite Sauces
Fresh
Garlic
ea. .9

Recommendation 4

1. Using color tiles show 3 red and 8 blue tiles. Create word problems involving the following ratios: 3 to 8 and 8 to 3 (part-to-part) and 3 to 11 and 8 to 11 (part-to-whole).
2. If the recipe calls for 1/2 of a cup of sugar to make a dozen cookies, how much sugar would be needed if we triple the recipe's ingredients?
3. If the scale indicates 1 cm = 150 miles, and the distance on the map measures 7.5 cm, about how many miles would we need to travel?
4. Create a ratio table for the following problem: Three boys (Juan, Seth and Jared) shared a number of stamps in the ratio of 3:5:7. If Seth received 45 stamps, how many more stamps did Jared receive than Juan?
5. If the Raptors made 21 of 30 shots in the first half and also shot 30 shots in the 2nd half of the game, how many shots did they make?

Recommendation 4

Think About

1. The three "cases" of % can all be represented using a proportion, including each of the following:
   - 68 is what percent of 80? (e.g. 68/80 = x/100)
   - What is 350% of 50?
   - 23% of 210 is what number?
   - 17 is what percent of 51?
   - 85% of what number is 102?
   - 200% of what number is 120
2. Can we think of proportion as an extension of equivalent fractions?
3. Rate is a special application of ratio. If we can buy 9 tickets for $54, how much does each ticket cost?
4. Common contexts: unit pricing, scaling (enlarge, reduce, map scales), recipes, mixtures.
5. Hundred grids (charts) may be more useful here than manipulative materials.
6. Connections to CCSS (Grades 6, 7)


QuickTime Player File Edit View Share Window Help
Stop Recording
(0:27) Sun 7:19 PM
Fractions tools & activities | Conceptua Math
http://www.conceptuamath.com/index.html
Google
Most Visited
Curricular Programs...
Leadership Issues M...
Getting Started
Latest Headlines
Conducting CFIP ~ S...
Refworks Web Based...
Loyola Notre Dame L...
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Fractions tools & activities | Co...
Conceptua™ Fractions – Teache...
conceptua MATH
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HOME ABOUT PRODUCT APPROACH TEACH YOUR WAY PURCHASE COMMUNITY SUPPORT
The premier online tool for fractions instruction.
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Free trial - 60 days! >
Try out the premium features and a "fraction" of our activities.
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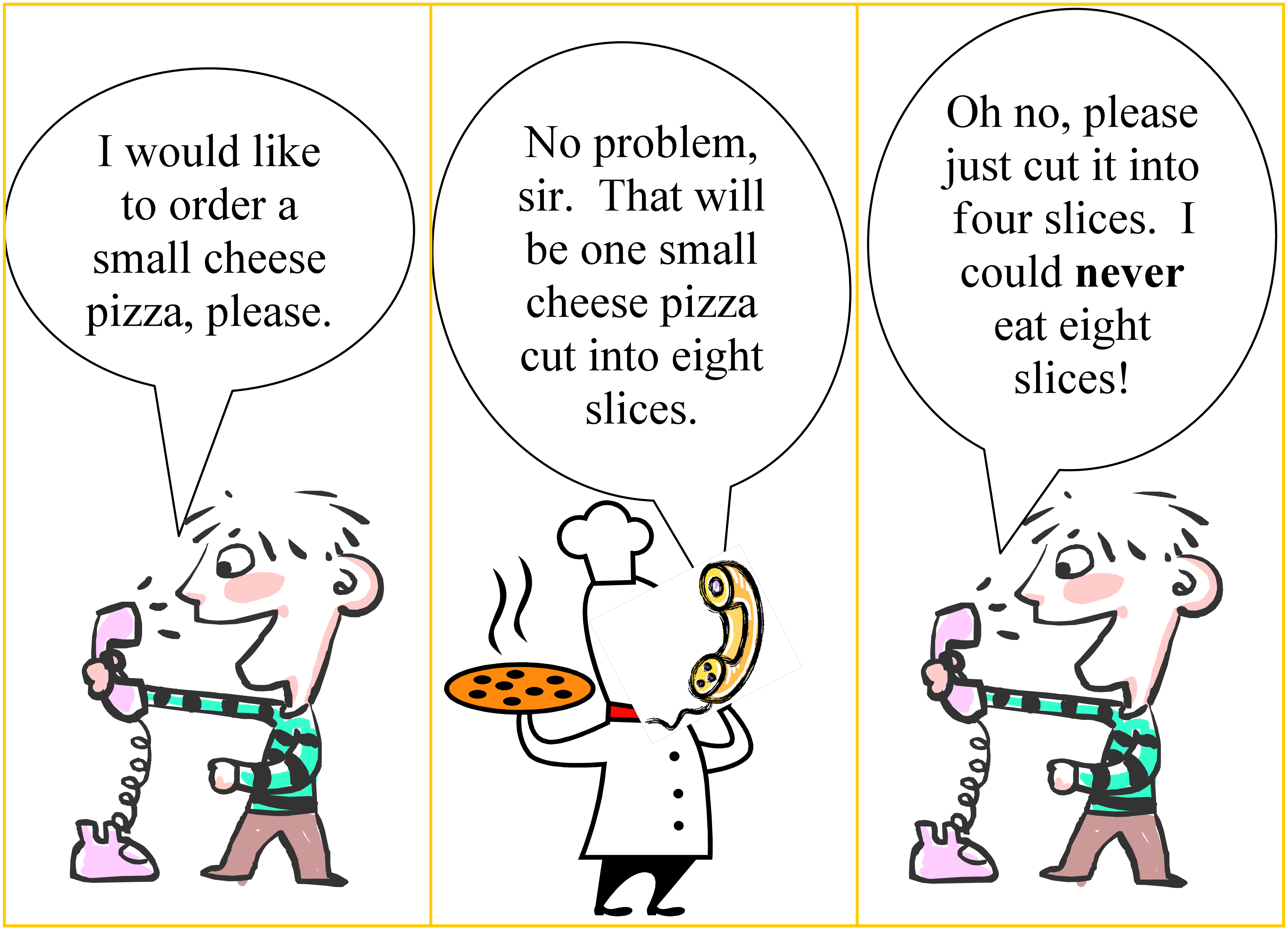
Ideal for whole-class instruction, these tools include fraction bars, circles, number lines and more.
Order your license >
Affordable subscription pricing for individuals, schools and districts.
Conceptua™ Fractions helps math teachers and students work together to master fractions. By combining interactive tools with scaffolded activities, students use visual models to build conceptual understanding and procedural fluency. Learn more >
Latest News
Texas-Sized Thank You!
Thanks to the hundreds of educators we met at TCEA. We loved meeting you! In addition to interest in the Free Tools and new Trial version, many teachers asked for a copy of our TEKS Alignment.
Done

Recommendation 5

Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them.

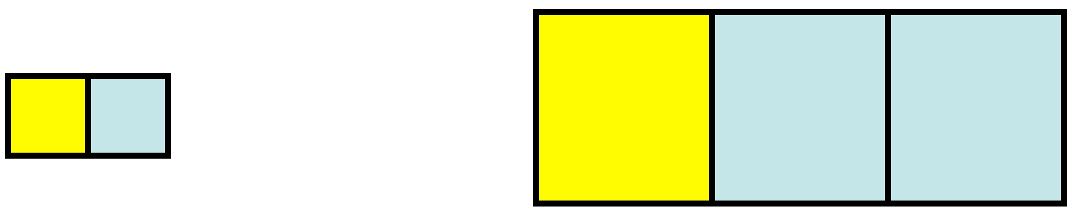
- Build teachers' depth of understanding of fractions and computational procedures involving fractions.
- Prepare teachers to use varied pictorial and concrete representations of fractions and fraction operations.
- Develop teachers' ability to assess students' understandings and misunderstandings of fractions.

We Still Have Work to Do!



Fraction beginnings…

- Which one is larger, 1/2 or 1/3?



“the size of the fractional part is relative to the size of the whole…” (NCTM, 2006)

Thinking about…

- $\frac{1}{2} \times \frac{1}{4} =$

- $\frac{1}{2} \div \frac{1}{4} =$

Your next steps…

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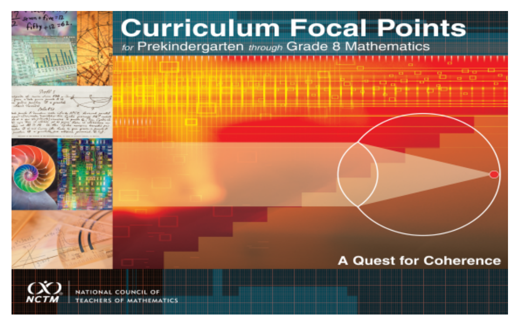
Focus and Coherence

- Informal Beginnings
  - Grades 1, 2
- **Number and Operations – Fractions**
  - **Grades 3-5**
- **Ratios and Proportional Reasoning**
  - **Grades 6, 7**
- The Number System
  - Grades 6, 7



Critical Areas

Grade 3

- *Developing understanding of fractions, especially unit fractions*

Grade 4

- *Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers*

Grade 5

- *Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)*

Grade 6

- *Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems;*
- *Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers*

Grade 3 – NO - Fractions

- Develop understanding of fractions as numbers.
  - 7

Grade 4 – NO - Fractions

- Extend understanding of fraction equivalence and ordering.
  - 2
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
  - 7
- Understand decimal notation for fractions and compare fractions.
  - 3

Grade 5 – NOBT

- Perform operations with multi-digit whole numbers and decimals to hundredths.
  - 1

Grade 5 – NO - Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
  - 2
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
  - 9

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Grade 6 – Ratios and Proportional Reasoning

- Understand ratio concepts and use ratio reasoning to solve problems.

Grade 6 – The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions
- Apply and extend previous understandings of numbers to the system of rational numbers.

Grade 7 – Ratios and Proportional Reasoning

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

Grade 7 – The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Survey

- Cell phone - *take it out!*

Concluding Thoughts

Recommendations

- Sharing and partitioning…;
- Fractions extend the number system (use this, CCSS);
- How procedures work and why;
- Applications – ratio, rate, and proportion
- Professional development needs – content and pedagogy